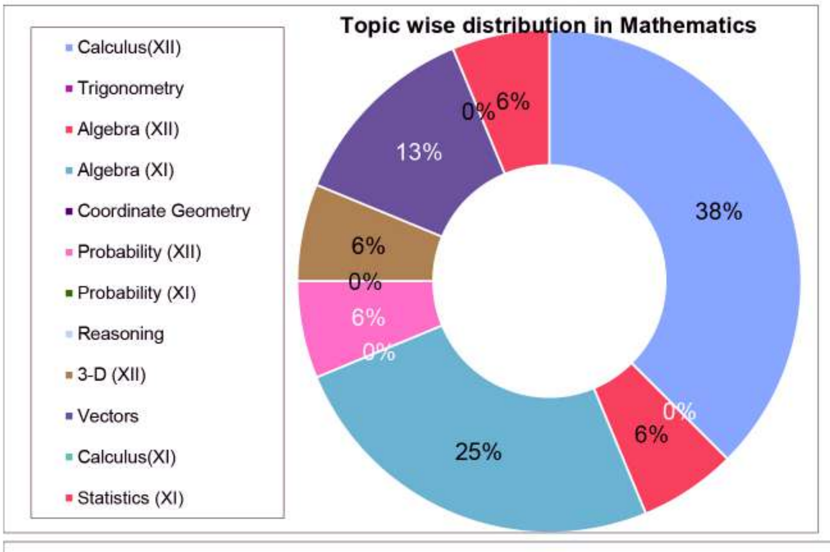
SINCE 1984

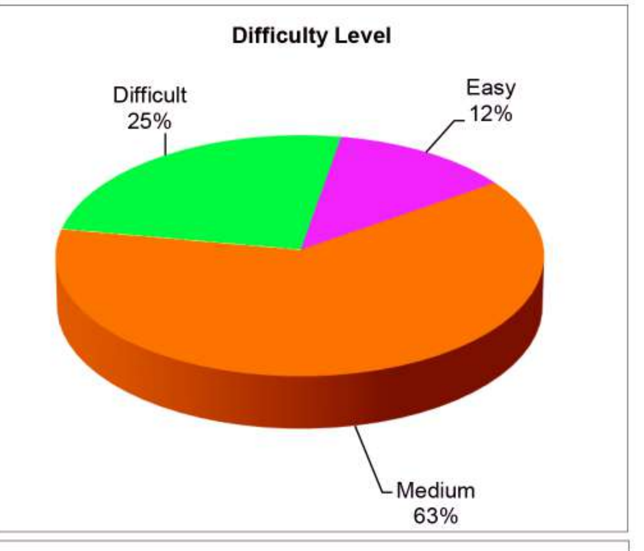
Brilliant STUDY CENTRE, PALA

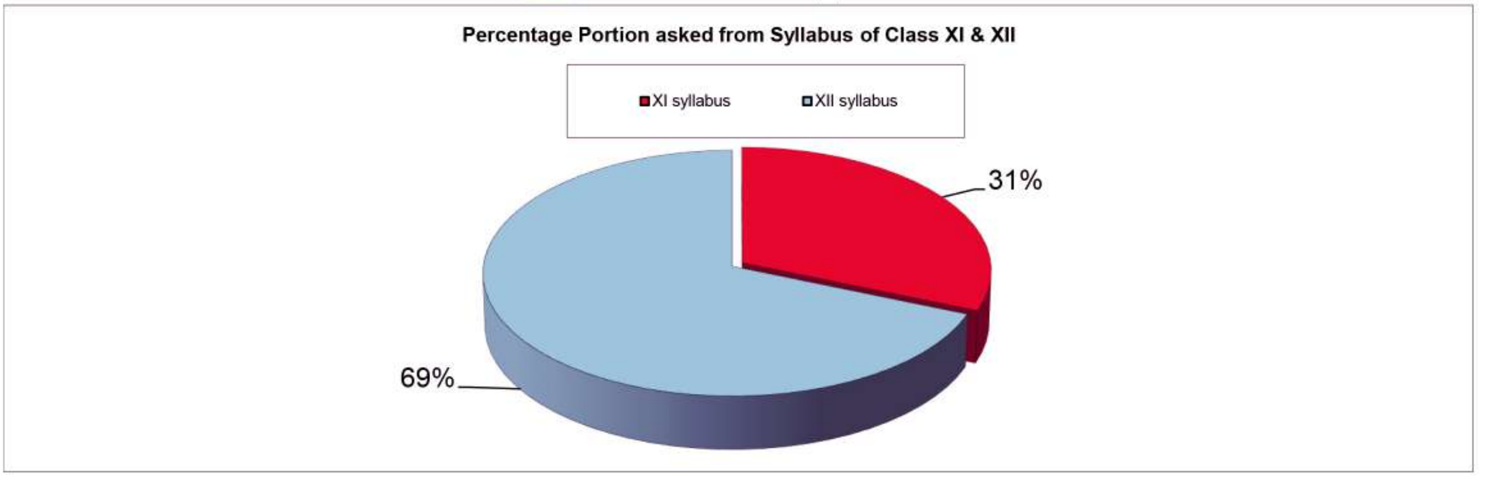
ANALYSIS OF JEE ADVANCED 2025 - MATHEMATICS PAPER-1

Topics		Easy	Medium	Difficult	*Total*	*Percentage*
Calculus(XII)	XII Class	0	4	2	6	37.50%
Trigonometry	XI Class	0	0	0	0	0.00%
Algebra (XII)	XII Class	0	0	1	1	6.25%
Algebra (XI)	XI Class	1	3	0	4	25.00%
Coordinate Geometry	XI Class	0	0	0	0	0.00%
Probability (XII)	XII Class	0	1	0	1	6.25%
Probability (XI)	XI Class	0	0	0	0	0.00%
Reasoning	XI Class	0	0	0	0	0.00%
3-D (XII)	XII Class	0	1	0	1	6.25%
Vectors	XII Class	1	0	1	2	12.50%
Calculus(XI)	XI Class	0	0	0	0	0.00%
Statistics (XI)	XI Class	0	1	0	1	6.25%
Total		**2**	**10**	**4**	**16**	**100%**

XII syllabus	**11**	**XI syllabus**	**5**







SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : $+3$ If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1 Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.
Define the functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$, and $h: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4,$$
$$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4,$$
$$h(x) = f(x+1) - g(x+2).$$

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(A)	8
(B)	2
(C)	-4
(D)	-6

Q.2 Three students S_1, S_2, and S_3 are given a problem to solve. Consider the following events:

U: At least one of S_1, S_2, and S_3 can solve the problem,
V: S_1 can solve the problem, given that neither S_2 nor S_3 can solve the problem,
W: S_2 can solve the problem and S_3 cannot solve the problem,
T: S_3 can solve the problem.

For any event E, let $P(E)$ denote the probability of E. If

$$P(U) = \frac{1}{2}, \quad P(V) = \frac{1}{10}, \quad \text{and} \quad P(W) = \frac{1}{12},$$

then $P(T)$ is equal to

(A)	$\frac{13}{36}$	(B)	$\frac{1}{3}$	(C)	$\frac{19}{60}$	(D)	$\frac{1}{4}$

Q.3 Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\dfrac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0\,. \end{cases}$$

Then which one of the following statements is TRUE?

(A)	The function f is **NOT** differentiable at $x = 0$
(B)	There is a positive real number δ, such that f is a decreasing function on the interval $(0, \delta)$
(C)	For any positive real number δ, the function f is **NOT** an increasing function on the interval $(-\delta, 0)$
(D)	$x = 0$ is a point of local minima of f

Q.4 Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3×3 invertible matrices Q with integer entries, such that

$$Q^{-1} = Q^T \quad \text{and} \quad PQ = QP\,,$$

is

(A)	32	(B)	8	(C)	16	(D)	24

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : $+4$ **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : $+1$ If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get $+4$ marks;
choosing ONLY (A) and (B) will get $+2$ marks;
choosing ONLY (A) and (D) will get $+2$ marks;
choosing ONLY (B) and (D) will get $+2$ marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get -2 marks.

Q.5 Let L_1 be the line of intersection of the planes given by the equations

$$2x + 3y + z = 4 \text{ and } x + 2y + z = 5 .$$

Let L_2 be the line passing through the point $P(2, -1, 3)$ and parallel to L_1. Let M denote the plane given by the equation

$$2x + y - 2z = 6 .$$

Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M .

Then which of the following statements is (are) TRUE?

(A)	The length of the line segment PQ is $9\sqrt{3}$
(B)	The length of the line segment QR is 15
(C)	The area of ΔPQR is $\frac{3}{2}\sqrt{234}$
(D)	The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Q.6 Let ℕ denote the set of all natural numbers, and ℤ denote the set of all integers. Consider the functions $f: \mathbb{N} \to \mathbb{Z}$ and $g: \mathbb{Z} \to \mathbb{N}$ defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd}, \\ (4-n)/2 & \text{if } n \text{ is even}, \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) TRUE?

(A)	$g \circ f$ is **NOT** one-one and $g \circ f$ is **NOT** onto
(B)	$f \circ g$ is **NOT** one-one but $f \circ g$ is onto
(C)	g is one-one and g is onto
(D)	f is **NOT** one-one but f is onto

Q.7 Let ℝ denote the set of all real numbers. Let $z_1 = 1 + 2i$ and $z_2 = 3i$ be two complex numbers, where $i = \sqrt{-1}$. Let

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : \ |x + iy - z_1| = 2|x + iy - z_2| \}.$$

Then which of the following statements is (are) TRUE?

(A)	S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$
(B)	S is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$
(C)	S is a circle with radius $\frac{\sqrt{2}}{3}$
(D)	S is a circle with radius $\frac{2\sqrt{2}}{3}$

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:
  Full Marks : $+4$ If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

Q.8 Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by $\mathcal{S}$.

Then the number of elements in $\mathcal{S}$ is __________________.

Q.9 For any two points M and N in the XY-plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle ΔPQR such that

$$\overrightarrow{SP} + 5\,\overrightarrow{SQ} + 6\,\overrightarrow{SR} = \vec{0}.$$

Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is ________________.

Q.10 Let S be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in S, but 0210222 is **NOT** in S.
Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to _____________.

Q.11 Let α and β be the real numbers such that

$$\lim_{x\to 0} \frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}\,dt + \beta x\cos x\right) = 2\,.$$

Then the value of $\alpha+\beta$ is ___________.

Q.12 Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(x)>0$ for all $x\in\mathbb{R}$, and $f(x+y)=f(x)f(y)$ for all $x,y\in\mathbb{R}$.

Let the real numbers $a_1, a_2, \dots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$, and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25}+1)\,,$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is ___________________.

Q.13 For all $x>0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx} - (\sin x)^2\, y_1 = 0,\quad y_1(1) = 5\,,$$
$$\frac{dy_2}{dx} - (\cos x)^2\, y_2 = 0,\quad y_2(1) = \frac{1}{3}\,,$$
$$\frac{dy_3}{dx} - \left(\frac{2-x^3}{x^3}\right) y_3 = 0,\quad y_3(1) = \frac{3}{5e}\,,$$

respectively. Then

$$\lim_{x\to 0^+} \frac{y_1(x)y_2(x)y_3(x) + 2x}{e^{3x}\sin x}$$

is equal to _______________.

SECTION 4 (Maximum Marks: 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

Q.14 Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	f_1	f_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
(P) $7f_1 + 9f_2$ is equal to	(1) 146
(Q) 19α is equal to	(2) 47
	(3) 48
(R) 19β is equal to	(4) 145
(S) $19\sigma^2$ is equal to	(5) 55

(A)	(P) ⟶ (5)	(Q) ⟶ (3)	(R) ⟶ (2)	(S) ⟶ (4)
(B)	(P) ⟶ (5)	(Q) ⟶ (2)	(R) ⟶ (3)	(S) ⟶ (1)
(C)	(P) ⟶ (5)	(Q) ⟶ (3)	(R) ⟶ (2)	(S) ⟶ (1)
(D)	(P) ⟶ (3)	(Q) ⟶ (2)	(R) ⟶ (5)	(S) ⟶ (4)

Q.15 Let $\mathbb{R}$ denote the set of all real numbers. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let n denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	**List-II**
(P) The minimum value of n for which the function $f(x) = \left[\dfrac{10x^3 - 45x^2 + 60x + 35}{n}\right]$ is continuous on the interval $[1, 2]$, is	(1) 8
	(2) 9
(Q) The minimum value of n for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is	
(R) The smallest natural number n which is greater than 5, such that $x = 3$ is a point of local minima of $h(x) = (x^2 - 9)^n(x^2 + 2x + 3)$, is	(3) 5
(S) Number of $x_0 \in \mathbb{R}$ such that $l(x) = \sum_{k=0}^{4}\left(\sin\lvert x-k\rvert + \cos\left\lvert x - k + \frac{1}{2}\right\rvert\right)$, $x \in \mathbb{R}$, is **NOT** differentiable at x_0, is	(4) 6
	(5) 10

(A)	(P) $\longrightarrow$ (1)	(Q) $\longrightarrow$ (3)	(R) $\longrightarrow$ (2)	(S) $\longrightarrow$ (5)
(B)	(P) $\longrightarrow$ (2)	(Q) $\longrightarrow$ (1)	(R) $\longrightarrow$ (4)	(S) $\longrightarrow$ (3)
(C)	(P) $\longrightarrow$ (5)	(Q) $\longrightarrow$ (1)	(R) $\longrightarrow$ (4)	(S) $\longrightarrow$ (3)
(D)	(P) $\longrightarrow$ (2)	(Q) $\longrightarrow$ (3)	(R) $\longrightarrow$ (1)	(S) $\longrightarrow$ (5)

Q.16 Let $\vec{w} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u} \times \vec{v} = \vec{w}$ and $\vec{v} \times \vec{w} = \vec{u}$. Let α, β, γ, and t be real numbers such that

$\vec{u} = \alpha\hat{\imath} + \beta\hat{\jmath} + \gamma\hat{k}$, $-t\,\alpha + \beta + \gamma = 0$, $\alpha - t\,\beta + \gamma = 0$, and $\alpha + \beta - t\,\gamma = 0$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	**List-II**
(P) $|\vec{v}|^2$ is equal to	(1) 0
(Q) If $\alpha = \sqrt{3}$, then γ^2 is equal to	(2) 1
(R) If $\alpha = \sqrt{3}$, then $(\beta + \gamma)^2$ is equal to	(3) 2
(S) If $\alpha = \sqrt{2}$, then $t + 3$ is equal to	(4) 3
	(5) 5

(A)	(P) ⟶ (2) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (5)
(B)	(P) ⟶ (2) (Q) ⟶ (4) (R) ⟶ (3) (S) ⟶ (5)
(C)	(P) ⟶ (2) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (3)
(D)	(P) ⟶ (5) (Q) ⟶ (4) (R) ⟶ (1) (S) ⟶ (3)

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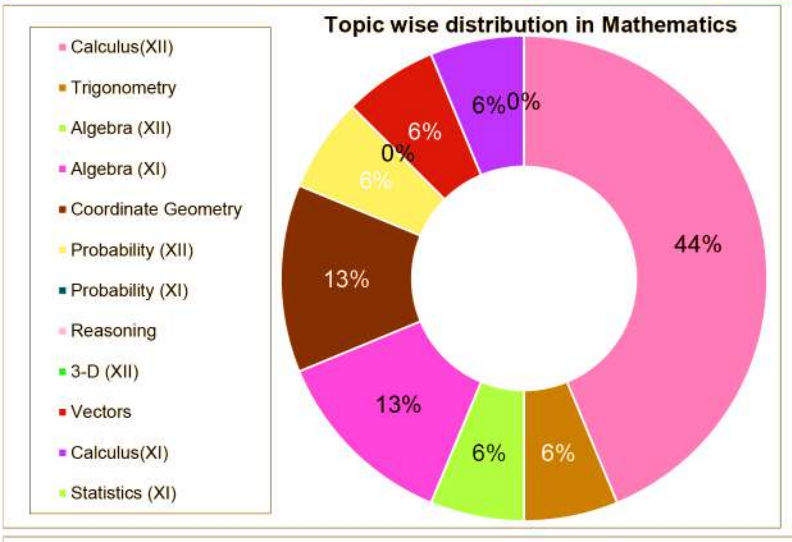
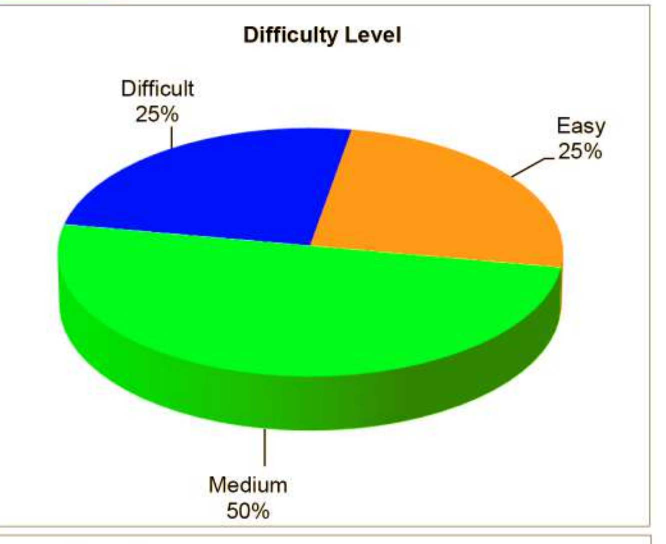
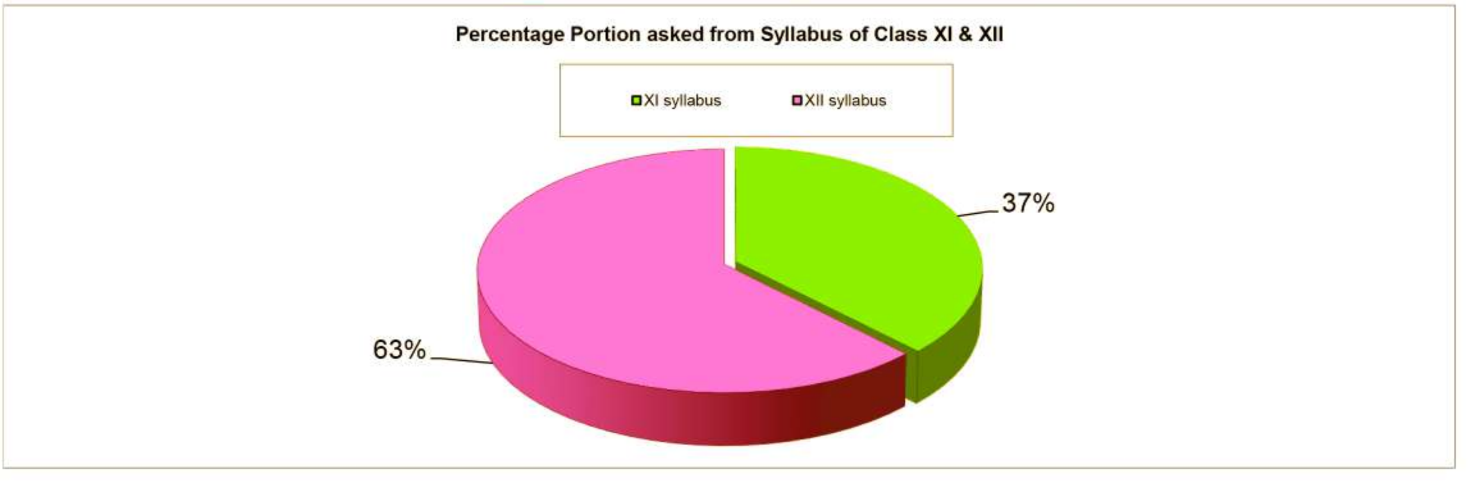
ANALYSIS OF JEE ADVANCED 2025 - MATHEMATICS PAPER-2

Topics		Easy	Medium	Difficult	*Total*	*Percentage*
Calculus(XII)	XII Class	1	4	2	7	43.75%
Trigonometry	XI Class	0	1	0	1	6.25%
Algebra (XII)	XII Class	0	0	1	1	6.25%
Algebra (XI)	XI Class	2	0	0	2	12.50%
Coordinate Geometry	XI Class	0	2	0	2	12.50%
Probability (XII)	XII Class	0	1	0	1	6.25%
Probability (XI)	XI Class	0	0	0	0	0.00%
Reasoning	XI Class	0	0	0	0	0.00%
3-D (XII)	XII Class	0	0	0	0	0.00%
Vectors	XII Class	1	0	0	1	6.25%
Calculus(XI)	XI Class	0	0	1	1	6.25%
Statistics (XI)	XI Class	0	0	0	0	0.00%
Total		***4***	***8***	***4***	***16***	**100%**

XII syllabus	**10**	**XI syllabus**	**6**







SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Q.1 Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers x.

Then which one of the following statements is TRUE?

(A)	For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$
(B)	For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$
(C)	For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$
(D)	For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

Q.2 Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0\right\}$$

is

(A)	$\frac{17}{16} - \log_e 4$	(B)	$\frac{33}{8} - \log_e 4$
(C)	$\frac{57}{8} - \log_e 4$	(D)	$\frac{17}{2} - \log_e 4$

Q.3 The total number of real solutions of the equation

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$$

is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively.)

(A)	1	(B)	2	(C)	3	(D)	5

Q.4 Let S denote the locus of the point of intersection of the pair of lines

$$4x - 3y = 12\alpha\,,$$
$$4\alpha x + 3\alpha y = 12\,,$$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points $(p, 0)$ and $(0, q)$, $q > 0$, and parallel to the line $4\,x - \frac{3}{\sqrt{2}}\,y = 0$.

Then the value of pq is

(A)	$-6\sqrt{2}$	(B)	$-3\sqrt{2}$	(C)	$-9\sqrt{2}$	(D)	$-12\sqrt{2}$

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : $+1$ If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get $+4$ marks;
  choosing ONLY (A) and (B) will get $+2$ marks;
  choosing ONLY (A) and (D) will get $+2$ marks;
  choosing ONLY (B) and (D) will get $+2$ marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -2 marks.

Q.5 Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y, and z, for which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$.

Then which of the following statements is (are) TRUE?

(A)	The determinant of $Q - 2I$ is zero
(B)	The determinant of $Q - 6I$ is 12
(C)	The determinant of $Q - 3I$ is 15
(D)	$yz = 2$

Q.6 Let S denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let $\mathcal{R}$ denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve S, and the lines $x = 1$ and $x = 4$.

Then which of the following statements is (are) TRUE?

(A)	$(4, \sqrt{3}) \in S$
(B)	$(5, \sqrt{2}) \in S$
(C)	Area of $\mathcal{R}$ is $\frac{14}{3} - 2\sqrt{3}$
(D)	Area of $\mathcal{R}$ is $\frac{14}{3} - \sqrt{3}$

Q.7 Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

such that $y_1 > 0$, and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point $(3, 0)$.

Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where O denotes the origin $(0, 0)$. Let $|XY|$ denote the length of the line segment XY.

Then which of the following statements is (are) TRUE?

(A)	The equation of the line joining P and Q is $2x + 3y = 3(1 + \sqrt{3})$
(B)	The equation of the line joining P and Q is $2x + y = 3(1 + \sqrt{3})$
(C)	If $N_2 = (x_2, 0)$, then $3\,|N_2Q| = 2\,|N_2S|$
(D)	If $N_1 = (x_1, 0)$, then $9\,|N_1P| = 4\,|N_1R|$

Q.8 Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{6x + \sin x}{2x + \sin x} & \text{if } x \neq 0, \\ \frac{7}{3} & \text{if } x = 0. \end{cases}$$

Then which of the following statements is (are) TRUE?

(A)	The point $x = 0$ is a point of local maxima of f
(B)	The point $x = 0$ is a point of local minima of f
(C)	Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3
(D)	Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

SECTION 3 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
Zero Marks : 0 In all other cases.

Q.9 Let $y(x)$ be the solution of the differential equation

$$x^2 \frac{dy}{dx} + xy = x^2 + y^2, \quad x > \frac{1}{e},$$

satisfying $y(1) = 0$. Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is ___________.

Q.10 Let $a_0, a_1, \dots, a_{23}$ be real numbers such that

$$\left(1 + \frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$

for every real number x. Let a_r be the largest among the numbers a_j for $0 \le j \le 23$.
Then the value of r is ___________.

Q.11 A factory has a total of three manufacturing units, $M_1, M_2,$ and M_3, which produce bulbs independent of each other. The units $M_1, M_2,$ and M_3 produce bulbs in the proportions of $2:2:1$, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by M_1, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$.

If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is ___________.

Q.12 Consider the vectors

$$\vec{x} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \qquad \vec{y} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \qquad \text{and} \qquad \vec{z} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}.$$

For two distinct positive real numbers α and β, define

$$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}, \qquad \vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}, \qquad \text{and} \qquad \vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}.$$

If the vectors $\vec{X}, \vec{Y},$ and $\vec{Z}$ lie in a plane, then the value of $\alpha + \beta - 3$ is_____________.

Q.13 For a non-zero complex number z, let $\arg(z)$ denote the principal argument of z, with $-\pi < \arg(z) \le \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right).$$

Then the value of $\frac{3\alpha}{\pi}$ is ________________.

Q.14 Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to (0, 4)$ be functions defined by

$$f(x) = \log_e(x^2 + 2x + 4), \text{ and } g(x) = \frac{4}{1 + e^{-2x}}.$$

Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = f\left(g^{-1}(x)\right)$, where g^{-1} is the inverse of the function g.

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is ______________.

Q.15 Let

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \cdots + \frac{1}{\sin 118^\circ \sin 119^\circ}.$$

Then the value of

$$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2$$

is ______________.

Q.16 If

$$\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2}\, dx,$$

then the value of $\sqrt{7} \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$ is _____________.

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)